DEFAULT REASONING AS A DECISION PROCESS ON INDUCTIVE INFERENCES

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ABSTRACT

Default reasoning has acquired importance in AI as a way to represent knowledge and common sense reasoning. After early fixed-point-based propositional entailment methods declined, classical approaches in the neat quarters has been to achieve nonmonotonicity in a syntactical manner. Dissent, however, has emphasized a decision theoretic point of view from which certain, perhaps limiting, results can be derived.

This paper follows the last approach, showing how Social Choice theory provides a unified framework where default reasoning methods can be embedded. Besides, in a topological analysis of the structure of a default theory, it will be shown that there exists two strata in such a theory, a deductive one and a default one.

The links of this approach with current conceptions about the nature of scientific theories is discussed in relation to possible implementations of the presented method.

I- Introduction:

One of the main goals of AI is to mechanize the reasoning process, and particulary the cognitive feature called *common sense* This goal is far from being attained, due to the lack of well suited formal methods to manage it. Historicaly this quest belong to Logic. But the limitative theorems proved in the 30´s by Gödel, Church and Turing convinced many that the philosophical rôle of Logic should be the study of methods able to verify the correctness of reasoning, that is, that of an a posteriori analytic.

Such a revision of the rôle of Logic in AI appeared as a critique of predicate calculus as a representation of common sense reasoning. The most criticized feature of formalized theories in first order languages was **monotonicity**. Therefore, a great deal of effort was dedicated to the research of nonmonotonic methods following a clear tendency: to modify FOL to handle nonmonotonicity inside the formalism [1]. This approach generated a number of different formalisms as **circumscription** [9], **default logic** [15], **nonmonotonic logic** [10] (and its embellished version, **autoepistemic logic** [11]), **the logics of change and action** [18], **the semantics of inheritance networks** [19], etc. To check conclusions, all these formalisms have to verify consistency of theories. This means that new inconvenients as the non validity of the classical metatheorems (deduction, consistency, etc.) are added to the traditional formal problems.

Many authors proposed alternative approaches, noteworthy the consideration of revision of theories not as a logical problem but as an epistemological one [6], and the rational handling of inferences (in a decision theoretic setting) [2]. One of the pourposes of this paper is to show how and why these heterodox criteria can be embedded in a game theoretical frame. The crucial step, in this direction is to show the evolution of theories as a **conclusion-choice process**. Theories stated in a logical language can be seen as "moves" in a game against *Reality* (the evidential context of theories). Added evidence to a theory is then "a move of Reality", followed by a reformulation of the theory as an answer.

With tools borrowed from the theory or Social Choice (a kind of decision theory with multiple criteria) it will be shown how it is possible and indeed more perspicuous to determine methods to mechanize the beforesaid game, and the effect of these methods on certain topological features of theories will be analyzed.

In section II, the structure of reasoning systems that handle default theories will be studied from an epistemological point of view. In section III an overview of the main features of Social Choice Theory will be given, related to the interpretation of default theories. Next section will be concerned with the topological structure of default theories as a help in the design of more robust methods. In section V a couple of examples are presented and in the last section the general conclusions and forthcoming work is discussed.

II - Default reasoning from an epistemological point of view:

In spite of a common structure, scientific theories are of a different kind that the strictly logical ones. There are two proper levels in every theory: the syntactical and the semantical ones, being the first the form of expression and the second the expressed concept. The first difference appears in the semantical aspect, because models in a scientific theory should be limited in number. This is due to the fact that the **meaning** of a theory of this kind depends not only from its syntactical structure (as in purely logical theories) but also, and mainly, from a third semiotic level: pragmatics (the relation of the theory with its users and with Reality).

The design of theories about any fragment of Reality has to take into account the three mentioned aspects, but recognizing syntax as (obviously) subordinated to semantics and pragmatics. This implies that if contextual information of the model is modified, the pragmatical relations between context and model will force a syntactical modification of the theory, consisting in a reformulation of its statements or in a relaxation of the grammatical constraints on its language.

A fundamental epistemological problem has been to define clearly how and when the syntax of a scientific theory has to be modified. A first proposal was given by K. Popper, who considered that the only syntactical feasible inferences in a theory are deductive (meaning that all the information needed to draw a conclusion has to be in its antecedents). Therefore, the negative contrastation of a theory has to be attributed to an **error**, forcing the withdraw of the theory and the design of a better one. That is to mantain the grammar and write new statements [14].

However, T. Kuhn, examining historical examples concluded that scientists faced to negative evidence reformulate their theories partially, and only when it is absolutelly impossible to follow this procedure, just then a new theory is designed [7]. This approach was followed by many authors adscribed to the so called *structuralist* school. They proposed formalizations for this process, characterized by distinguishing several strata of stability in a theory. The most stable stratum is the hard core of the theory, which in case of not being correct forces the failure of **all** the theory. Within this conception, a theory posseses several models corresponding to all feasible modifications. Any theory in this view, shows no equality between its **intensional** and **extensional** contents, so being **nonmonotonic**.

Despite its conceptual clarity, postkuhnian epistemology is still susceptible to the remark that considering only **deductive inferences** obscures the truly tentative nature of a scientific theory. Besides, while the popperian dynamic of theories obeys clearly the "all or nothing" feature of deductive logic, the postkuhnian script appears as a set of *ad hoc* rules without a deeper theorical justification.

Several proposals in AI for formalizing default reasoning are analogous in nature to postkuhnian epistemology, due to the obsesive fixation to deductive inference (perhaps because it is the most neatly formalized reasoning method available). However a slight evaluation of reasoning forms shows that besides the deductive one exist the inductive, the analogical, and the abductive or conjectural ones, being the last two the most important for scientifical thought and common sense. All the forms of reasoning, except deduction, are characterized by allowing to draw conclusions without enough information. But between them are remarkable differences, the main one being that only inductive inference can be stated through rules in which consequences are drawn tentatively from antecedents. Therefore, in case of common sense reasoning, inference rules can be considered **default rules**.

Inductive reasoning was always associated to probability theory [14], but in case of being considered as default rules acquires some distinctive features. Default rules were taxonomized in the following classes: **prototypical** (involving generic plural statements), **individual** (concerning normal states of objects, facts or situations) and **systemic** (concerning states of information relevant for a set of default rules) [2]. The crucial point here lies in that although the **vocabulary** of default theories is classical, their **grammar** is not so, and therefore they can be "inconsistent" in a deductive sense. Then, actual modification of a default theory occurs in its model. An unsuitability of this approach is that, lacking methods for handling conjectural reasoning (able to allow the design of new default rules up from a set of evidences), the only permissible procedure is to shorten the range of rules through the explicitation of exceptions. This is, of course, a failure common to all known default reasoning formalisms.

Given the computational frame of AI, a consideration on the implementation of a methodology to handle the evolution of default theories is in order. The point is to distinguish the four levels that should appear in any implementation:

a) Default rules (perhaps together with classical ones).
b) A database of facts and tentative conclusions.
c) A source of new evidence.
d) A mechanism of revision of theories.

It is worth noting that alterations required in d) are over the set defined in a), even though the direction almost pursued in current research is to modify the set defined in b). It should be clear that the most complicated part in the design of an implementation is the definition of the mechanism of revision of theories. This task requires an epistemological approach to provide a well grounded method.

III - Reconstruction of theories as a decision-theoretic process:

Are we constrained to attack the problems above only from the syntactical or the semantical side? Our intended answer is that pragmatic stances are more perspicuous and abarcative (the higher the better). In the proposed framework every default rule determines a bias in the model preference, which can be in turn restated by new incorporated evidence. Doyle's proposal is to see default rules as rational preference indicators [2], each one being a decision-theoretic criterion between alternative conclusions of a theory. This is enterely in accordance with the conception of default rules as inductive statements interpreted like rational decision rules [8] in the frame of subjective probability [14].

Contradictions between conclusions and evidences indicates the emergence of new decision criteria, because new evidence can be seen as default rules without antecedents. The conflict between criteria forces to choose one of them to mantain consistency in the set of conclusions.

This situation is similar to the votation problem: candidates are the conclusions of the theory and default rules the voters. Determination of the set of chosen conclusions is then the main problem for the modification of a default theory. This apparently trivial problem is a fundamental one from an epistemological standpoint. Inductive (and default) theories are inconsistent if considered deductive. Therefore, models involving tentative conclusions cannot be obtained but with a refereeing between conflicting defaults. Features of this decision process determine the kind of default theories to be obtained, raising a multiplicity of possible models for the same set of proper rules (like scientific theories in the postkuhnian view). Votation is the paradigmatic problem of Social Choice theory. As it name indicates, Social Choice theory tries to determine the different ways in which the wills of several individuals in social aggregates can be combined to give place to a "social will".

In Social Choice theory, given a set of individuals G, it is required that each individual g should have a preferential ordering R(g) of the countable set of social states X in such a way that from the individual orderings a unique ordering R can be obtained for all G.

The problem is thus to determine R, given the individual R(g). K. Arrow proved that the following conditions (for a democratical determination of R) are incompatible, where P and I are relations of strict preference and of indiference, respectively, and apostrophes indicate alternative orderings:

1) *Positive responsiveness*

For any x,y in X, if for all g, xP(g)y--->xP'(g)y
and xI(g)y--->xR'(g)y
and for some g xI(g)y and xP'(g)y
then xRy--->xP'y

2) *Pareto principle*

if xR(g)y for all g then xRy

3) *Non dictatorship*

It doesn't exist g such that for all x,y
if xR(g)y then xRy

4) *Independence of irrelevant alternatives*

If for all g exist R(g) and R'(g) and for all
x, y in X, xR(g)y<--->xR'(g)y then R = R'

5) *Transitivity*

for all x,y,z in X if xRy and yRz then xRz

As it will be shown, incompatibility between these constraints is very important for the determination of the set of conclusions of a default theory, given the interpretation of X as the set of conclusions sets of the theory G, each g as a default rule and R(g) an ordering of sets of conclusions according to g.

Doyle includes an additional constraint, the **principle of Paretian Optimality**:

if xRy and yR(g)x for some g then exists g' such that xR(g')y

and proves that the **stability** of default theories (the existence of closed and complete extensions of them [14]) implies the Paretian optimality [2]. It is, however, enough to consider only the classical constraints 1 to 5, because Paretian optimality can be derived in a relaxed version of them, as it will be shown.

Incompatibility between constraints 1 to 5 is usually interpreted as indicating that a dictator has to exist (Arrow's theorem is also called the *dictator theorem*). To avoid this result condition 5 can be relaxed to provide the following constraint:

5') *Quasitransitivity*

R is quasitransitive iff for x,y,z in X, xPy and yPz imply xPz

In this case it is proved that a relation R exist, satisfying 1 to 5' [4]. This relation can be characterized as follows:

xPy iff for all g xR(g)y and there exists g' such that xP(g')y

This definition is, in fact, the classical statement of the principle of Paretian optimality [5] and therefore (assuming the equivalence of stability of theories and Paretian optimality, as proved by Doyle) that it is capable, when applied to default theories, to find stable sets of conclusions.

The proposed method of default theories interpretation is abarcative: if as an alternative method is considered one involving only rules 1 to 4, obeying Pareto principle, equivalent results to circumscription, skeptical inheritance, or CWA will be provided [2]. As the general method obeys also the Pareto principle, it is clear that these conservative forms of non default reasoning can be embedded in the forms handling stable theories. The only form of default reasoning not included, considering constraints 1 to 5' is the represented by **production rules**, (applied *sequentially*). This is of course equivalent to a principle of sequencial dictatorship.

Implementation of this method requires every default rule to have a preferential ordering on the subsets of conclusions. This order can be obtained simply by set inclusion, considering the inclusion of grounded conclusions of the rule in the subsets. It should be noted that the only rules defining a strict ordering are the classical ones, because default rules are purely tentative. Finally, taking the global order for the theory, the maximal set in this order is the new conclusions set of the theory, that is **the new model**. It is clear that although no formal complexity analysis has been performed, given the exponential growing of the power set of the set of conclusions due to the inclusion of new evidence, this method has NP complexity. Topology of ordered sets comes to the rescue, as it is shown in the next section.

IV - Topology of default theories.

In the last section was shown how, with a reduced number of conditions it is possible to establish a method for the reformulation of default theories in face of new evidence. It is clear that evidence will shorten the range of default rules, and unless a method to create new rules taking into account the limitations impossed by the growing information (a "fix" in the sense of Perlis [13]) is added, only the axioms and the conclusions derived through classical rules will remain in the conclusions set.

In KR, as in epistemology, fixes are undesiderable because they are scfuffy, *ad hoc* and procedural, and rules continuosly incorporating exceptions in their body prevent better reformulations of the theory. The same can be said about default theories, which can be seen as sets of preferences over tentative conclusions. The fixes, providing immunity against errors (through a very poor kind of learning) are in this context also condemnable. Thus, it is appropiate to analyze the form in which the body of rules contributes to the collapse of the theory, providing a taxonomy of stability strata, instead of trying to repair the damages caused by contradictions.

The key to determine this strata of stability in default theories is to analyze the topological structure determined by the proposed methods of reasoning, equivalent to well known default reasoning formalisms. The tools are again provided by Social Choice Theory.

An important notion, that allows to examine in detail the action of default rules as indicators of preference on subsets of conclusions, is that of **decisive group** [17] defined over a pair of alternatives x, y of X as a set of rules D(x,y) such that:

xRy iff xR(g)y for all g in D(x,y)

Evaluation of the topological structure defined by the methods of determination of R will be carried on the class O of decisive groups of rules to find out which subsets of rules will be more significative for the model of the theory when new evidence is incorporated.

To acomplish this goal lets consider some well known results on the topological properties of decisive sets. The notions of **filter**, **prefilter** and **ultrafilter** are the core of this topological analysis. Given two sets T and O, being the later a subset of the power set of the former that obeys the following constraints:

a- T pertains to O
b- If a set G pertains to O, and G is included in a set J, then J pertains to O
c- If the sets G_i are included in O for i=1..n then the intersection of the G_i is not empty
d- If G and J pertain to O, then their intersection pertains to O
e- If G doesn't pertain to O, its complement T-G does

O is a **prefilter** if satisfies a, b and c. If O satisfies also d it is called a **filter** and if it verifies all the properties it is an **ultrafilter**. It has been proved that:

- The Pareto principle (the method of determination of R satisfies 1 to 4) implies that O is a prefilter.

- Pareto optimality (the method satisfies 1 to 5') implies that O is a filter.

- Dictatorship (the method verifies 1 to 5) implies that O is an ultrafilter.

The first result can be interpreted as stating that there are a subset of T, not pertaining to O, such that each one of its elements pertains to each set of O, belonging to a *Collegium* whose elements have decision power over every pair of possible alternatives. In the case of Paretian optimality the second result states that the Collegium becomes a decisive set (an Oligarchy). The last result means simply that the oligarchy has only one element.

Relating these results with the stated in the last section, it is obvious that if T is the set of rules of a default theory (including classical rules) the more conservative methods of default reasoning will be characterized by having a Collegium, that is a set of rules that will be decisive individually for each conclusion in which they participate, but not constituying a "hard core" of the theory. The more courageous methods of reasoning have such a core, instead.

Given that only deductive rules determine strict preferences, it is clear that they will the exclusive members from the Collegium or the Oligarchy. This result shows that there are only two strata in a default theory: deductive rules(including axioms) and default rules.

It can be speculated that proper default rules can form also a many sorted set, given the different relevance of rules. Formalizations of this idea require to rank the default rules as a first step to perform reasoning, as it is acknowledged in [3].

V - Examples:

As an example of this method lets consider the case of Opus the penguin. The theory consists of the following rules:

I) penguin(x) ===> bird(x)

II) bird(x) ---> flies(x)

III) penguin(x) ===> -flies(x)

IV) penguin(Opus)

where ===> represents "strict" deductive implication and ---> represents "weak" default implication. The sets of maximaly consistent conclusions are:

A= {penguin(Opus), bird(Opus), flies(Opus)}

B= {penguin(Opus), bird(Opus), -flies(Opus)}

To find the global set of conclusions. consider the preference relations between A and B given by rules I to IV:

AI(I)B , AR(II)B , BP(III)A , AI(IV)B

In this case, Paretian optimality provides:

BRA

That means that B is prefered to A, due to axiom III, the only one that can determine a strict preference relation. Therefore, the new set of conclusions includes the evidence of Opus not flying and excludes the tentative but wrong conclusion that it does. Considering, as another example, the famous "Nixon diamond", where the rules are:

I) Republican(x)---->-Pacifist(x)

II) Quaker(x)------->Pacifist(x)

Given the evidence that Nixon is Republican and Quaker the corresponding axioms are:

III) Republican(Nixon)

IV) Quaker(Nixon)

Then the conclusions set plus the evidence will be:

{Republican(Nixon),Quaker(Nixon),Pacifist(Nixon),-Pacifist(Nixon)}

To avoid inconsistency and lacking additional information, the method of Paretian optimality will consider only subsets without contradictions. The sets to be considered will be A = {Republican(Nixon), Quaker(Nixon), Pacifist(Nixon)} and B = {Republican(Nixon), Quaker(Nixon), -Pacifist(Nixon)}. Then the following relations can be valid:

BR(I)A, AR(II)B, AI(III)B, AI(IV)B

Given the tie score between A and B the method can only conclude:

AIB

which is an expectable result knowing the equivalence of Paretian Optimality and theories with stable extensions this method cannot decide for one of the sets. Considering the method based in the Pareto principle only one set is prefered, because it is the only weakly prefered by all rules:

{Quaker(Nixon),Republican(Nixon)}

which is another expectable result considering the equivalence between the Pareto principle and skeptical inheritance.

VI - Conclusions:

Another rationale for coping with default reasoning has been presented in this paper, considering defaults as a kind of inductive rules and, emphasizing the tentativity of infered conclusions. Besides, a notion of model of default theories (inductive theories with elements of commonsense) with an epistemological bent was examined as preferable to a logical one.

To formalize the beforesaid concepts several tools borrowed from Social Choice theory have been useful. Two main results have been derived:

1) All forms of default reasoning can be seen as succesive constraints to an universal method of interpretation of default theories.

2) The set of classical rules (including evidences) is the decisive group on the conclusions of default theories.

Result 2 indicates that the methods of default reasoning are less "robust" than the scientific theories, as seen by the structuralist school of epistemology: given that this postkuhnian view admits in every theory several strata of stability, more than two kinds of formulations are decisive to the inferred conclusions. Perhaps an approach as considered in [16] may cope with the acknowledged problem of the ranking between defaults.

A way to strengh default theories is to use probabilistic semantics as proposed in [12]. Given that Decision Theory provided the tools to obtain both results of this paper, it is useful to look in it for other means to solve this problem. An attractive notion is that of expected utility, that allows to evaluate the relative importance of rules with respect to new evidence, and which also deserve further research.

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VII - References:

[1] Bobrow, D.
Editor's Preface
Artificial Intelligence 13:1,2 (1980).

[2] Doyle, J.
On universal theories of defaults
Technical Report CMU-CS-88-111 (1988)

[3] Geffner, H.
Default Reasoning: Causal and Conditional Theories
Ph. D. Thesis.UCLA TR 137, (1989)

[4] Heal, G.
The theory of economic planning
Amsterdam, North-Holland (1973)

[5] Intriligator, M.
Mathematical optimization and economic theory,
N.Y., Prentice-Hall (1971).

[6] Israel, D.
What's wrong with non monotonic logic?
Proceedings of the First National Conference on AI (1980).

[7] Kuhn, T.
The structure of scientific revolutions
Chicago, The University of Chicago Press, (1962).

[8] Luce, R.- Raiffa H.
Games and decisions
N.Y., Wiley&Sons (1957).

[9] McCarthy, J.
Circumscription, a form of nonmonotonic reasoning
Artificial Intelligence 13:1, 2 (1980).

[10] McDermott, D.- Doyle, J.
Nonmonotonic logic I
Artificial Intelligence 13:1,2 (1980).

[11] Moore, R.
Semantical considerations on nonmonotonic logic
Artificial Intelligence 25, 1(1985)

[12] Pearl, J.
Probabilistic semantics for default reasoning
Technical Report. UCLA-CSD-870052 (1987).

[13] Perlis, D.
On the consistency of commonsense reasoning
Comput. Intell. 2 (1986).

[14] Popper, K.
The Logic of Scientific Discovery
London, Hutchinson & Co., (1959)

[15] Reiter, R.
A logic for default reasoning
Artificial Intelligence 13:1,2 (1980).

[16] Rescher, N.
Plausible reasoning
Amsterdam, Van Gorcum, (1976)

[17] Sen, A.
Social choice theory
in *Handbook of Mathematical Economics*
Arrow, K.-Intriligator, M. eds., N.Y., Elsevier (1986).

[18] Shoham, Y.
Reasoning about change: time and causation from the standpoint of Artificial Intelligence
Yale Univ. Ph.D. thesis (1986).

[19] Touretzky, D.- Horty, J.- Thomason, R.
A clash of intuitions: the current state of nonmonotonic multiple inheritance systems
Proceedings IJCAI-87 (1987).